$\vec{F}$ is conservative in a simply-connected region of interest

Zero rest length spring: $\vec{F} = K(\vec{r}_2 - \vec{r}_1)$

$\vec{F}(t, \vec{r}, \vec{v}) = \vec{F}(\vec{r})$

Near Earth Gravity
Far Earth gravity
Springs

Example: $\vec{F} = -Kr\hat{e}_r = -K(x\hat{i} + y\hat{j})$

Example $\vec{F} = Kr\hat{e}_\theta = K(-y\hat{i} + x\hat{j})$

$\vec{F}$ is conservative

1.
$E_p(\vec{r})$ exists with $\vec{F} = -\vec{\nabla} E_p$ $\quad F_x = -\frac{\partial E_p}{\partial x}$, $F_y = -\frac{\partial E_p}{\partial y}$

Ex.1. $E_p = \frac{Kr^2}{2}$, E_p = no such thing

curl
2. $\vec{\nabla} \times \vec{F} = 0$ $\quad \frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x}$

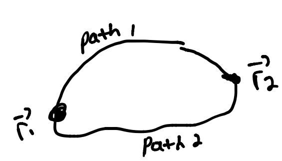
Ex.1 $0 = 0$ ✓

Ex.2 $-1 \neq 1$ X

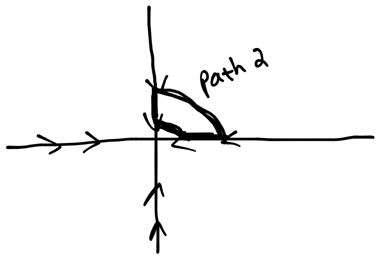
3. For any pair, $\vec{r}_1$ and $\vec{r}_2$ and any pair of paths between them:

$\int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r}$ is path independent

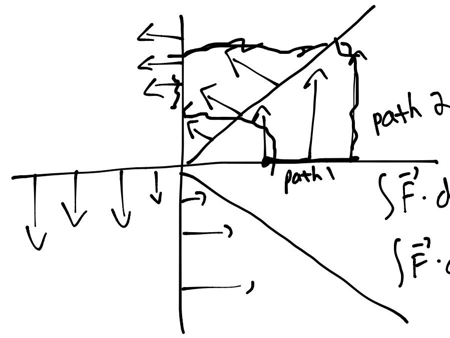
The Work integral



Ex.1



Ex 2



$\int \vec{F}\cdot d\vec{r} \neq 0$

$\int \vec{F}\cdot d\vec{r} = 0$

4. $\oint \vec{F}\cdot d\vec{r} = 0$ all closed paths

"net work of the force when going on a 'round trip' = 0"

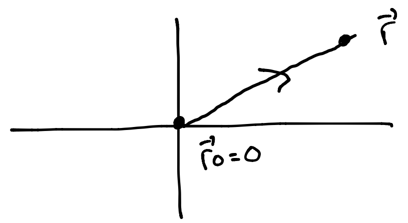
Ex.1 $\oint \vec{F}\cdot d\vec{r} = 0$ Ex 2. $\oint \vec{F}\cdot d\vec{r} \neq 0$

If one closed path is nonzero, its bad

5. $E_p(\vec{r}) = \int_{\vec{r}_0}^{\vec{r}} \vec{F}\cdot d\vec{r}$ $\vec{r}$ of your choice, path of your choice

Ex.1 $\vec{F} = K\vec{r}$

$\int_{\vec{r}_0}^{\vec{r}} \vec{F}(\vec{r})\cdot d\vec{r}\,'$



parameterization: $\vec{r}\,' = t\vec{r}$

$0 \leq t \leq 1$

$d\vec{r}\,' = d(t\vec{r}) = \vec{r}\,dt$

$\vec{F}(\vec{r}) = K\vec{r}\,' = Kt\vec{r}$

$\vec{F}(\vec{r}\,')\cdot d\vec{r}\,' = Kt\vec{r}\cdot\vec{r}\,dt$

$= r^2 Kt\,dt$

$$-\int_{\vec{r}_0}^{\vec{r}} \vec{F}\cdot d\vec{r}' = \int_0^1 kr^2 t\,dt = -kr^2 \int_0^1 t\,dt = -\frac{kr^2}{2} = E_p$$

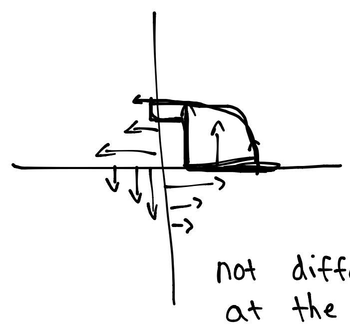
Ex.2 There is no E_p

$\vec{F} = -y\hat{i} + x\hat{j}$ Non-conservative valuable

Subtle example $\vec{F} = \frac{1}{r}\hat{e}_\theta$

conservative or not conservative

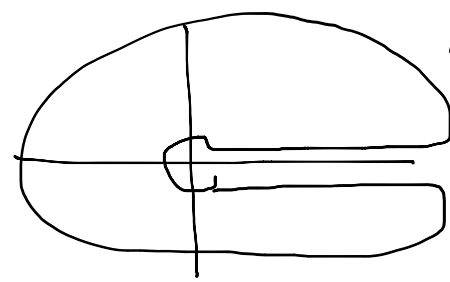
$\vec{\nabla} \times \vec{F} = 0$ everywhere but O



not differentiable at the origin

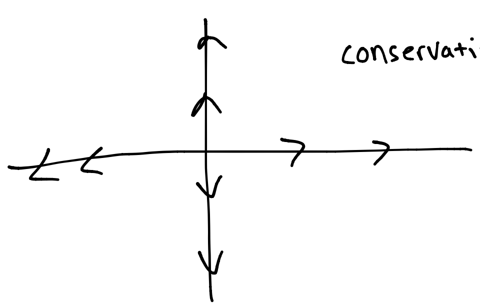
You can't go around the origin

- if you do it's not conservative



conservative

Ex $\vec{F} = k\vec{r}$



conservative, and goes to infinity

negative spring

Multi-particle systems

$$\vec{F} = \vec{F}(t, \vec{r}^1, \vec{r}^2 \ldots \vec{r}^i \ldots \vec{r}^n, \vec{v}^1, \vec{v}^2 \ldots \vec{v}^n)$$